EXPERIMENTAL TESTBEDDING OF A POLLUTION TRADING SYSTEM: SOUTHERN CALIFORNIA'S *RECLAIM* EMISSIONS MARKET

Takashi Ishikida, John Ledyard, Mark Olson and
David Porter

I. INTRODUCTION

The efficient management of air quality through the use of pollution emissions trading systems is not a new idea (see Montgomery (1972)). However, the implementation of such systems is new. The design of the pollution permits being exchanged and the market in which the permits are traded requires the integration of environmental economics, game theory, operations research and experimental testing (see Ledyard (1993)). This chapter reports on experiments that were used to help design and testbed a novel pollution trading system used in Southern California. This system allows participants to trade two pollutants (nitrogen and sulfur oxides – NOx and SOx) across two zones (upwind and downwind) over 9 years separate years (1994–2003).[1] The complexity associated with such an interdependent system of commodities makes the design of the trading system challenging. Testbedding new systems is standard fare in engineering but only recently has been applied to economics through the use of experiments (see Plott (1994)). Unlike experiments designed to test specific theories of behavior, testbedding is used when theory supplies little design advice and when the process is relatively new and there is no experience

---

Research in Experimental Economics, Volume 8, pages 185–220.

ISBN: 0-7623-0702-1

with its operation. In this chapter, we describe the experimental research we utilized in designing, developing, testing and engineering a two-sided iterative combinatorial market for the Regional Clean Air Incentives Market program (RECLAIM) that established property rights in emission credits in Southern California.

The basic economic features of RECLAIM are: (1) the market has very few participants (there are 390 NOx-producing facilities and 41 SOx-producing facilities, but only a handful have participated in the market so far); (2) there is no established trading history so that price discovery is *not* a low cost activity; (3) regulatory uncertainty that stems from potential future rule changes, based on political pressure, can affect the value of emission credits.[2] These features of the economic environment describe what financial economists would call a *thin market*. Thin markets can create problems for standard market designs such as broker-dealer markets because the thinness results in larger bid-ask spreads and poor price discovery properties (see Campbell et al. (1991)). In general, when markets are thin, order aggregation through call markets tends to make for better price discovery methods and reduced price volatility (see McCabe et al. (1993)).

The most successful emission trading market to date has been the EPA's SO_2 allowance market. In Title IV of the Clean Air Act Amendments of 1990, U.S. electric utilities were required to reduce sulfur dioxide (SO_2) emissions by approximately ten million tons from 1980 levels. The Amendments specify annual caps on total SO_2 emissions with each regulated utility receiving an allocation of SO_2 allowances each year.[3] The Amendments also permit the creation of markets to buy and sell allowances. The allowances are *bankable* so that unused allowances could be carried forward to future years. However, the EPA auction designed for the SO_2 allowance trading has some poor incentive properties for the bidders. In particular, Cason and Plott (1996) show that the EPA's allowance auction causes participants to strategically underreveal their willingness-to-pay and willingness-to-sell. Recently, Joskow et al. (1998) questioned the experimental and theoretical results of Cason (1995) and Cason and Plott (1996) because the private markets that trade in SO_2 allowances provide participants in the EPA auction with information about the marginal value of allowances. In addition, they suggest that empirical results from the auction do not support a cost underrevalation by sellers. Nonetheless, Joskow et al. (1999) do acknowledge that the SO_2 markets are far from the idealized frictionless transmitters of information given the uncertainty and long-term nature of abatement investments.

The argument over the performance of the SO_2 market is not the focus of this paper. Furthermore, the RECLAIM program differs significantly from the SO_2

program in several ways. First, allowances in RECLAIM are designated by areas so that trading credits from one area to another is not allowed. Second, there is no banking of credits in RECLAIM. Lastly, credits are also designated by compliance cycles and these cycles overlap.

The predecessor of RECLAIM was the Emission Reduction Credit Market (ERC). This program involves limited trading of credits that are generated when facilities shut down, retrofit or can show real sustained emission reductions from their source. This was a broker market with sequential bilateral trading. However, sequential and bilateral ERC trades between non-contiguous sources implies that air quality increases in one location and decreases in the other which can violate standards imposed by State Implementation Plans. In order to obtain true market equilibrium, simultaneous trading of permits by "all" firms that takes into account location ambient standards is required. The ERC market was not designed in this manner. Transaction costs of searching for sellers and ensuring the feasibility of an ERC trade has resulted in large commission fees and few trades (see Atkinson and Tietenberg (1991)).

In the remainder of this study we show how the market for RECLAIM credits was designed to counteract market-thinning agents inherent in standard market designs.[4] In the next section we provide some background information on the RECLAIM program in relation to the trading market and then describe the general structure of the market.

II. BRIEF DESCRIPTION OF THE RECLAIM PROGRAM

On October 15, 1993, the South Coast Air Quality Management District, the geographical area around Los Angeles, California, adopted RECLAIM. RECLAIM is a market-based program of air pollution incentive rules that mandates reductions in emissions of nitrogen oxide (NOx) and sulfur oxide (SOx) by imposing factory mass emissions caps. Under RECLAIM, a manufacturer is given an annual emissions limit for his entire plant, and yearly reductions to the plant-wide limit. Specifically, all sources at a particular facility are collectively assigned annual emissions cap based on historic rates of a selected three-year period. The cap defines the amount of emissions the facility is allowed for the year. The cap declines annually (8.3% per year on average for NOx and 6.8% per year on average for SOx) from 1994 to 2003. The program applies to the largest emission sources for these pollutants in the Los Angeles Basin.

Facilities receive RECLAIM Trading Credits (RTCs) equal to their annual caps; a facility emitting less than its allocation cap is permitted to sell its excess emissions to other facilities in the form of RTCs. If a factory finds that it will emit more than its cap, it may purchase RTCs from other facilities or make reductions in emissions by installing control equipment or reducing production. An RTC is designated by the year, compliance cycle, the pollutant (NOx or SOx), and its zone. The first *compliance cycle* runs from January 1 to December 31 of each year and the second cycle runs from June 1 to July 31 of each year. *Zones* refer to Los Anlgeles Basin geography and prevailing offshore winds. The zones restrict transfers so that *upwind* sources cannot use purchased *downwind* RTCs beyond their caps. Each RTC is valid for the single year cycle and zone for which it was issued, i.e. banking is not allowed. RECLAIM facilities that violate their annual emissions cap risk losing their factory permit and are subject to a maximum penalty of $10,000 per day. Unlike the SO_2 program, RECLAIM did not establish or specify a mechanism for the trading of RTCs; it only established a marketable asset or property right.

In the design of a market to trade RTCs, several features of the economic environment caused us to focus our attention on combinatorial call markets. These features are:

(1) there are a total of 124 different RTCs that can be substitutes, complements or constraints;
(2) buyers have demands for multiple RTCs of fixed size to meet production runs;
(3) there are demand synergies that arise from pollution abatement equipment economies of scale and scope;
(4) there are non-convexities that arise from the purchase of RTCs versus investment in abatement equipment that requires pricing a stream (or portfolio) of RTCs.[5]

Thus, the market must allow a small number of traders to submit bids and offers for multiple dimension packages of goods, restrict the feasible set of packages so that they meet the physical constraints of the bidder, and ensure that markets clear simultaneously so that price discovery is complete. This is no small task.

III. MECHANISM DESIGN AND INITIAL EXPERIMENT

The general market conditions that will be the focus of our design process are:

(1) The RECLAIM market is thin with a small number of participants;
(2) There is a need only to trade at quarterly compliance intervals when facilities can update their emission plans;

(3) There is no history of market trades; and

(4) Buyers and sellers would like to transact by selling whole portfolios of credits (all or nothing trading, economies of scale, complementary pollution emissions requirements, etc.)

Our first step was to consider the order aggregation process. Since not much information is available about the value of RTCs, we decided to focus our inquiry on *call market* designs in which exchanges occur at the same time so that all markets clear simultaneously.[6] In what follows we will describe two types of call auctions that could be used to trade RTCs. One design does not allow for combinatorial orders and has a single uniform price for each RTC. The other design allows for combinatorial orders and tries to preserve the uniform price structure when possible but allows for non-linear pricing.

Besides the call feature employed in each of the designs, we add the following:

(i) The markets are *Iterative*. Specifically, the markets are *not* one-shot. Traders can update bids and offers based on previous iteration results. This feature has been shown to be important in price discovery (see McCabe et al. (1993) and Banks et al. (1989)).

(ii) Bids and offers must be *Improvements*. To provide proper incentives for price discovery, a new bid, in order to be accepted, must improve on current bids (increase bid price) or offers (reduce offer price) by a certain percentage.

(iii) *Price* is calculated from marginal trades. Thus, inframarginal bids and offers have an incentive to reveal. Furthermore, whenever they exist, uniform competitive equilibrium prices are utilized so that price conveys information about scarcity.

First we examine a standard call market, the Uniform Price Double Auction (UPDA – see Smith et al. (1982), Van Boening (1991) and Davis and Holt (1993)). In this design each buyer i submits a set of bids for each commodity j of the form:

$$<(P^1_{ij}, Q^1_{ij}), \dots, (P^m_{ij}, Q^m_{ij})> = B_{ij} \tag{1}$$

where P^w_{ij} denotes the bid price in order w by buyer i for commodity j and Q^w_{ij} is the corresponding maximum quantity demanded.[7] Each seller k submits offers of the form:

$$<(C^1_{kj}, S^1_{kj}), \dots, (C^n_{kj}, S^n_{kj})> = O_{kj} \tag{2}$$

where C^v_{kj} denotes the offer price in order v by seller k for commodity j and S^v_{kj} is the corresponding maximum quantity offered (the same restriction in footnote 7 applies to the sellers).

Once these bids and offers are submitted, the center calculates the accepted bids and offers by solving the following linear program:

$$\mathrm{Max}_{a,b}\, \Sigma_i\, \Sigma_j\, \Sigma_r\, a^r_{ij} \bullet P^r_{ij} - \Sigma_k\, \Sigma_j\, \Sigma_g\, b^g_{kj} \bullet C^g_{kj} \tag{3}$$

such that

$$\begin{array}{ll} a^r_{ij},\, b^g_{kj} \in [0,1] & \forall \quad i, k, j, r, g \\ \Sigma_i\, \Sigma_r\, a^r_{ij} \bullet Q^r_{ij} - \Sigma_k\, \Sigma_g\, b^g_{kj} \bullet S^g_{kj} = 0 & \forall \quad j \end{array}$$

This is nothing more than finding the largest surplus such that supply equals demand for each commodity. The price paid by the "winners" is uniform for each j and is equal to:

$$p_j = (p^L_j + p^H_j)/2 \tag{4}$$

where

$$p^L_j = \begin{cases} \mathrm{Max}\ \{P^r_{ij}\text{: } a^r_{ij} = 0 \text{ and } P^r_{ij} > C^g_{kj} \text{ for } b^g_{kj} > 0\} & \text{(\textit{highest rejected bid}) if it exists} \\ p^H_j & \text{otherwise} \end{cases}$$

$$p^H_j = \begin{cases} \mathrm{Min}\ \{C^g_{kj}\text{: } b^g_{kj} = 0 \text{ and } P^r_{ij} > C^g_{kj} \text{ for } a^r_{ij} > 0\} & \text{(\textit{lowest rejected offer}) if it exists} \\ p^L_j & \text{otherwise} \end{cases}$$

if neither p^L_j nor p^H_j exists as defined above, we use:

$$\begin{aligned} p^L_j &= \mathrm{Min}\ \{P^r_{ij}\text{: } a^r_{ij} > 0\} \\ p^H_j &= \mathrm{Max}\ \{C^g_{kj}\text{: } b^r_{kj} > 0\} \end{aligned}$$

UPDA has been found to be comparable in efficiency to the double auction (see Smith et al. (1982), Liu (1992) and Friedman (1993)). In addition, since UPDA produces a single uniform price for each commodity, it lacks the price volatility of the double auction. The bidders have less opportunity to be influential on the final price, and there is less of a need to be strategic, except for those bids at the margin that influence the price.

An iterative version of UPDA would have at each iteration t, participants submitting bids and offers, B^t_{ij} and O^t_{kj}. Given these bids we calculate (3) and (4) to obtain solutions ${}^t a^{r*}_{ij}$, ${}^t b^{g*}_{ij}$ and ${}^t p^*_j$. For iteration $t+1$ the following restrictions hold:

$$\{B^t_i\text{: for } {}^t a^{r*}_{ij} > 0\} \subseteq B^{t+1}_{ij} \quad \forall i \ \text{ and } \ \{O^t_{kj}\text{: for } {}^t b^{g*}_{kj} > 0\} \subseteq O^{t+1}_{kj} \quad \forall k \tag{5}$$

This constraint says that all winning bids/offers in the previous iteration must be submitted in the next iteration (this a commitment constraint). In addition, the following restrictions are imposed:

$$\begin{aligned}&\text{For each bid in } \{B_i^t\text{: for } {}^t a_{ij}^{r*} > 0\}\ {}^{t+1}P_{ij}^{r*} = \alpha\, {}^tP_{ij}^{r*} \text{ where } \alpha > 1 \text{ and}\\ &\text{for each offer in } \{O_k^t\text{: for } {}^t b_{kj}^{g*} > 0\}\ {}^{t+1}C_{kj}^{g*} = \beta\, {}^tC_{kj}^{g*} \text{ where } 0 < \beta < 1\end{aligned} \qquad (6)$$

This constraint says that the winning bids (offers) in the previous iteration will be afforded a premium (discount). This is a reward for providing *price discovery* information and is an incentive to participate early in the market. The market stops when there is no improvement in the allocation (surplus).

To see how UPDA can fail to make surplus enhancing trade when market orders can have "all or nothing" constraints, consider the supply and demand condition depicted in Fig. 1. In this market there are three buyers and three sellers. Buyer B1 has a block demand (all or nothing order) for 5 units and a maximum willingness to pay of \$70 per unit, i.e. $V_1(x) = 350$ if $x \geq 5$ and $V_1(x) = 0$ otherwise, where V denotes the payoff function. Buyer B2 has a demand such that $V_2(x) = 40{*}x$ for $x \leq 2$ and $V_3(x) = 35{*}x$ for $x \leq 2$. Seller S1 has a cost of $C_1(x) = 50{*}x$ for $x \leq 3$, seller S2 has a cost $C_2(x) = 160$ if $x \leq 2$ and $C_2(x) = \infty$ if $x > 2$ and $C_3(x) = 90{*}x$ for $x \leq 2$. There is positive surplus in the market since buyer surplus for 5 units is 350 and sellers' cost for 5 units is 310. However, the maximum price that can be charged for 5 units is \$70; but at that price seller 2 has a cost of \$80 per unit. For RECLAIM, many of the buyers and sellers are faced with these types of "*all or nothing*" situations. When contemplating adding new abatement equipment, reductions come not in continuous units.

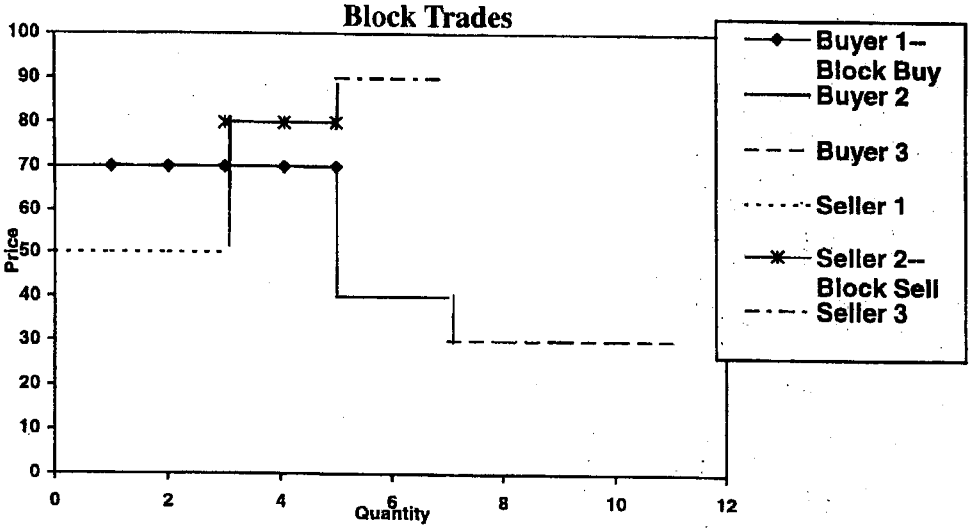


Fig. 1. All or nothing trading example where there is no uniform price equilibrium. However, there is surplus to participants if prices are allowed to be non-uniform.

When a facility wants to expand production runs, the decisions are not flexible in quantity demanded. While there is no uniform price equilibrium for the situation charted in Fig. 1, there is positive surplus if trade was to occur. If seller 2 were to receive $80 per unit for his two units, seller 1 received $55 for his three units and buyer 1 pays $65 per unit for 5 units, then the budget would be balanced and surplus attained. This also demonstrates that block orders may be able to obtain strategic rents.

In addition to all or nothing constraints there are other forms of demand that may cause problems for an UPDA type market. For example, suppose each market has a good x, whose payoff function V(x) is *superadditive* for participants, i.e. $V_i'(x) \geq 0$. These values can arise because demanders have economies of scale in their facilities or require a block of trading permits across their facilities. Suppliers in these markets have their costs induced via *opportunity cost*. That is, some participants have permits that they can use to satisfy their own pollution demands or can sell them to others.

Consider for example the three markets (these are labeled A, B and C) depicted in Fig, 2. There are eight participants in these markets. In Markets A and C, each step represents a participant's marginal payoff for the first three units. The marginal payoff for more than 3 units is zero. The payoff or demands for each participant is upward sloping. For example, participant 1 has a payoff of $20 for 1 unit of the good; his second unit supplies an additional payoff of $35 for a total of $55; his third unit provides an additional payoff of $45 for a total payoff of $100 if he obtains 3 units of the good. In Market A, there is no depth in the market since there are no extra marginal traders and there is a wide competitive price tunnel ($13–$25). In Market C, 10 units are offer for sale to participants in the market.[8] In Market C there is no uniform price equilibrium. This occurs because at prices below $46.6 participant 4 will demand 3 units (he has a value of 140 for three units). Thus there is an excess demand at those prices. As soon as the price goes above $46.6 participant 4 will demand zero units and there will be an excess supply. Thus the only outcomes that can occur under a uniform price condition are either the outcome must result in losses or at least one bidder must forgo the pursuit of potentially profitable opportunities. In Market B, there are 10 potential units for sale. Each of the 4 potential demanders has superadditive values for up to 10 units. Thus, the most efficient outcome has one demander obtaining all the units from the sellers. There is a uniform price competitive equilibrium in this market at a price of $22 (this is the average profit of the first rejected demander for 10 units). Values for these markets can be found in Appendix A.

To determine if an iterative UPDA could provide for efficient outcomes in this environment we conducted a series of experiments. Subjects were recruited

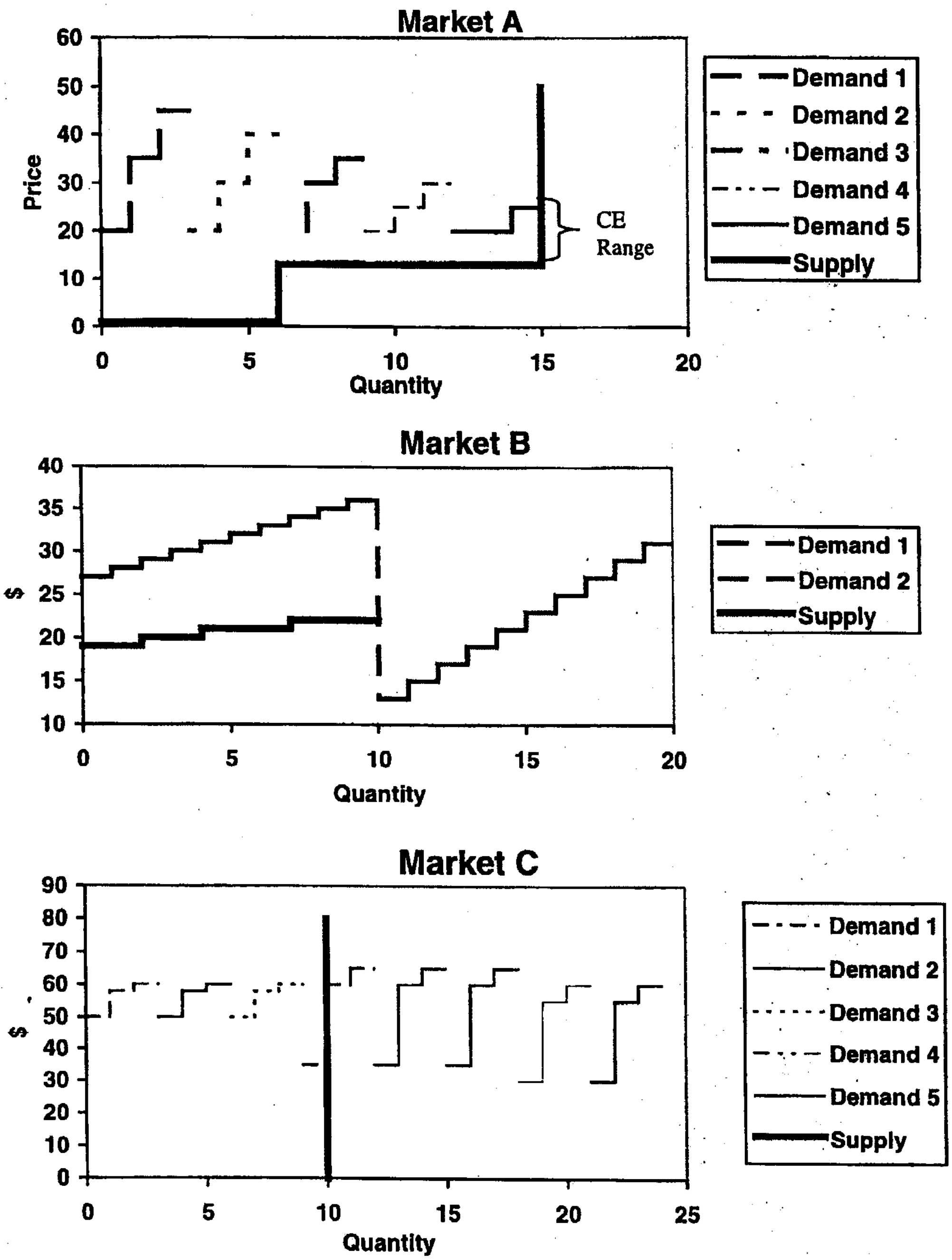


Fig. 2. Superadditive valued markets. Markets A and B have a uniform price competitive equilibrium price. Market C has a nonlinear competitive price equilibrium.

from the undergraduate student population at the California Institute of Technology. All the subjects had experience in other market experiments. Each experimental session consisted of a single market instance. Each session was run on a computer network and lasted approximately 1.5 hours. Each experimental session consisted of 3 commodities that were traded simultaneously. Subjects were assigned randomly to a set of redemption values, costs and endowments. Subjects were paid in cash at the end of the sessions.

Communication among subjects was not allowed during the experiment. While each subject knew her own valuations, costs, and endowments, she knew nothing about the valuations, costs, and endowments of the other subjects. Each subject knew the number of subjects in the experiment.

Each experimental session consisted of a single market period consisting of a sequence of iterations. Each subject had approximately 10 minutes to enter orders and was limited to a maximum of 30 orders in each iteration. At the beginning of each iteration, the current market price and the number of units tentatively accepted for trading in each market were displayed on subject computer terminals.

Two sessions were conducted with these environments. The volumes, prices and efficiencies for each market are provided in Fig. 3. For each Market, the price, volume and efficiency for each iteration of the auction are provided in the figures. Each session lasted 7 iterations before the markets closed. For Market A each session produced allocations that were at the 80% level of efficiency when the auction closed at iteration 7. Volume was significantly below the prediction of 15 units. Thus, UPDA did not do a good job of trading the marginal units. The same result occurs in Market B but volume of trade was slightly better. However in Market C efficiency was almost 100%. Even though this Market had no competitive equilibrium, participants managed to keep prices below the danger level of \$35.[9]

Besides the superadditive cases that derive from economies of scale in pollution abatement technologies, there are conditions in which there are strong complements between the pollutants. That is, a facility produces both pollutants when it has production runs. Thus, if the facility demands more pollution credits, it may require them in complementary proportions. Suppose for example, that bidders in Markets A and B must obtain fixed combinations of units of A and B in order to obtain value. That is, $V_i(x_A, x_B) = 0$ if $\{x_A, x_B\} \not\subset X_i$ where X_i is a finite set of combinations that person i values. The values for V_i and X_i for the results we provide below can be found in Appendix A. Figure 4 shows the outcomes in these complementary market environments. In session 1, the auction lasted 8 iterations before it closed. The prices, volumes and efficiencies for the complementary markets for each iteration are provided.

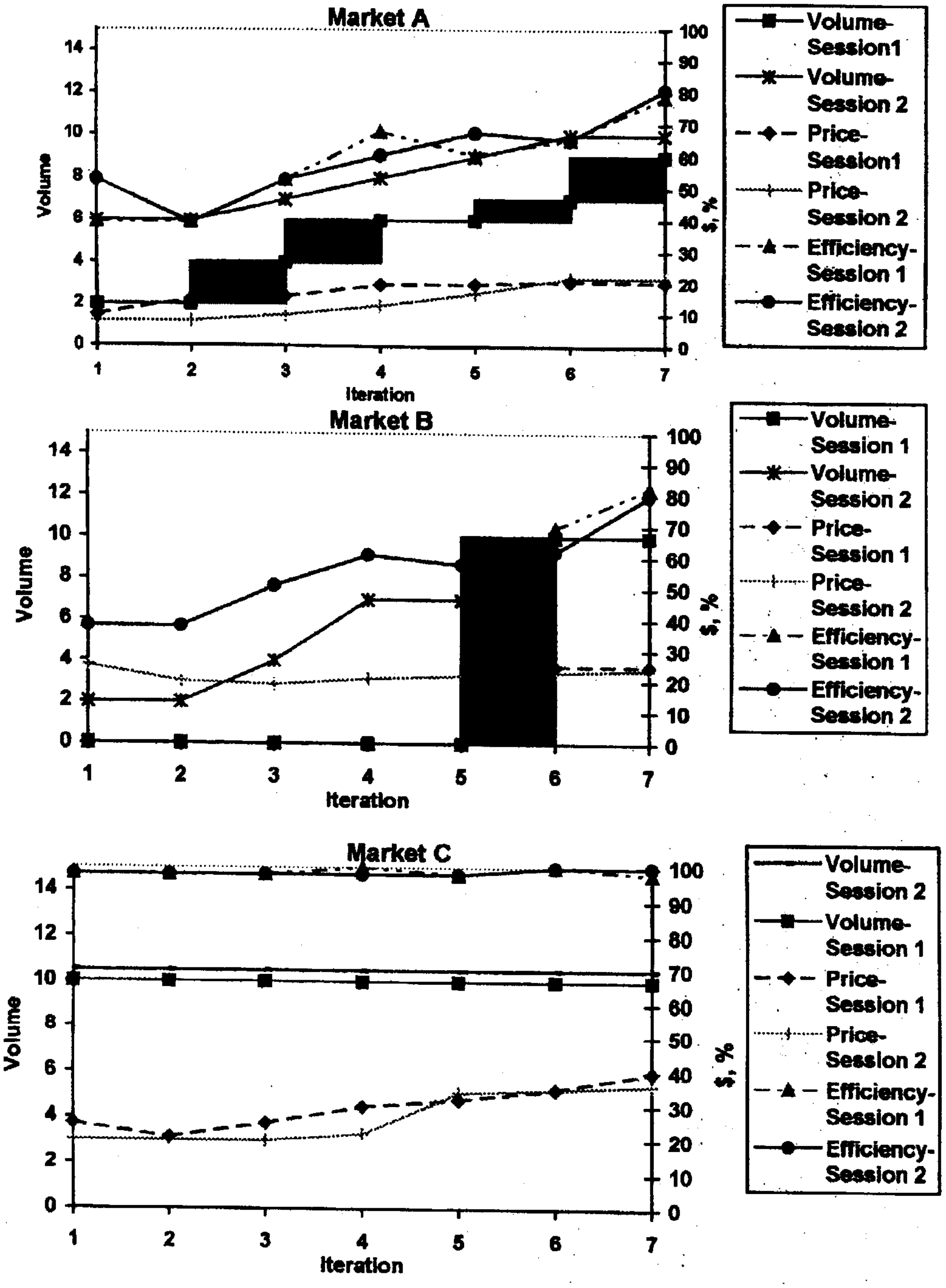


Fig. 3. Superadditive results using UPDA.

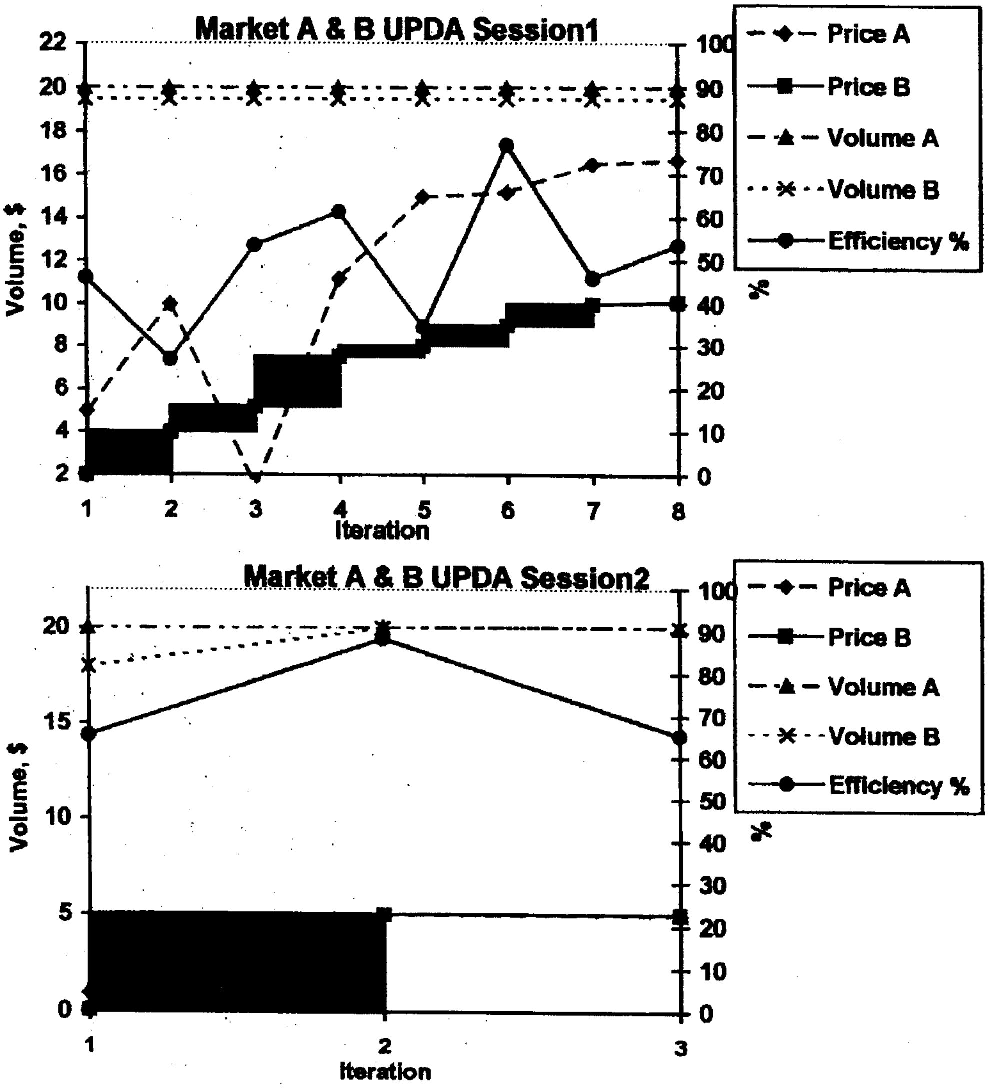


Fig. 4. Strong complements results using UPDA.

Session 1 produced a mere 53% efficiency. In session 2 which lasted only 3 iterations before the auction closed with a closing efficiency of 66%. In addition to the low efficiencies, the prices generated in the Markets were significantly different between the sessions.

It is clear that a standard market design does not generate fully efficient allocations. We now discuss the market design that we used to solve these non-convex demand structures.

IV. TWO-SIDED COMBINATORIAL MARKET DESIGN

When preferences have non-convexities, the use of combinatorial mechanisms can eliminate trades that can produce losses or the forfeiting of profitable opportunities because of the risk of financial exposure. For example, if a bidder requires a package of emission credits but must assemble them piecemeal, they may obtain only a subset of the package. Since there are different values for the package versus its parts, the proper bids for the piecemeal orders are indeterminate.

To reduce this source of uncertainty, we began a design exercise to create a mechanism that allows buyers and sellers to tailor their orders to increase efficiency and prevent financial losses in nonconvex environments. Our basic design goals were:

(1) to select and complete the set of trades that maximizes traders' true surplus;
(2) to charge everyone a price that leaves them at least as well off as if they did not participate; this is seen as "fair" by the participants (one part of fair is that everyone pays the same per unit price if that is possible); and
(3) to ensure that the auctioneer does not have to spend any of their money or extract surplus from the traders.[10]

If it exists, a linear competitive equilibrium will satisfy these conditions.

The general structure of this market parallels the iterative UPDA. In particular, at each iteration agents submit buy/sell orders; a tentative allocation is made that maximizes surplus (the sum of seller and buyer surplus). Winning orders are automatically entered into the next iteration and can be replaced only if a new allocation improves surplus. Bidders can modify rejected bids in any manner but they are not automatically entered in the next iteration. If surplus does not increase by 5% after iteration 2, the market ends and transactions are made based on the allocation of the last iteration.[11]

Conceptually the allocation problem takes submitted bids and maximizes surplus given demand and supply constraints and contingent constraints. This is a mixed-integer linear program (MILP) and there are many techniques to solve this type of problem (see Skiena (1997)). With 124 different RTCs and a large number of bids, this may not be computable in a reasonable length of time. For our initial market we broke the RTCs into four smaller markets: NOx zone 1, NOx zone 2, SOx zone 1, SOx zone 2; each one of these submarkets has 33 RTCs. In addition, there were restrictions placed on bids that retained the packaging and contingent capabilities but also reduced the computational burden.

There are four immediate problems: (1) What is the structure of allowable bids? (2) How do we determine which bids and offers to accept, and how do we accomplish this in a reasonable amount of time? (3) What information is given to the agents between iterations? (4) How are prices computed and what do the agents pay?

Given the potential complexity of the bids and offers, it will be important to provide clear and concise market information between iterations so that participants can successfully update their bids and offers. We will describe the allocation and pricing procedures later in this section, but first we will begin by describing the types of orders that are permissible in the market. We use the generic term *order* for bids to buy, offers to sell, and any packaged combination.

A. *Types of Orders*

Our combinatorial market allows for a variety of order types. A *multi-market order* for agent i is a vector $<b_i, q_{i1}, \ldots, q_{in}, F_i>$ where:

- $b_i > 0$ means agent i is willing to pay at most b_i for the order,
- $b_i < 0$ means agent i is willing to accept at least b_i for the order.
- $q_i > 0$ means agent i wants to purchase up to q_{ij} units of j in the order,
- $q_{ij} < 0$ means agent i wants to sell up to q_{ij} units of j in the order,
- F_i is a scale factor $(0 \leq F_i \leq 1)$ which indicates that agent i is willing to accept any one order of the form $<f_i \cdot b_i, f_i \cdot q_{i1}, \ldots, f_i \cdot q_{in}, >$ where $f_i \in [F_i, 1]$.

For example, an order of the form $<1000,(0,0,10),0.6>$ indicates that the agent is willing to purchase any size order between the full size and 60% of the full order. That is, he is willing to pay up to $1000 for 10 units of asset 3, or he is also willing to pay up to $600 for 6 units of asset 3 or any convex combination of these orders. If $F_i = 1$, we say that the agent's order is *inflexible*, and if $F_i = 0$ then we say the agent is *flexible*. As we will see, inflexibility will come at a cost to the agent.

In addition to multi-market orders, orders may be packaged, that is, an order may consist of a set of linked orders. For example, a packaged order can specify a willingness to pay up to $100 for 10 units of asset 2, pay $50 for 20 units of asset 3 *and* 30 units of asset 4. A packaged order may also consist of a *swap*, which consists of buying units of some assets and selling units of other assets. For example, a swap order can specify a willingness to pay up to $10 and supply 10 units of asset 4 *if and only if* 10 units of asset 2 are received.

Orders are also allowed to be connected by logical ORs (also called *contingent bids*). An OR is a logical element that binds one or more orders together. For example, an agent is able to submit an order that states a willingness to purchase 10 units of asset 3 for up to $10 *if and only if* they do *not* purchase 20 units of asset 4 for up to $20 and vice versa. This is equivalent to saying that out of set M orders, at most only one of the orders should be accepted.

We will now use i to denote orders as opposed to agent identification to reduce the notational burdens. The allocation at each iteration is determined by solving the integer program:

$$V^* = \underset{d_i}{\text{Maximize}} \ \Sigma_i \, b_i \bullet d_i \tag{7}$$

subject to

$$d_i = 0 \text{ or } F_i < d_i \leq 1 \qquad \textit{feasibility} \tag{8}$$

$$\Sigma_i \, d_i \bullet q_{ij} \leq 0 \ \forall j \qquad \textit{no excess demand} \tag{9}$$

$$\Sigma_{k \in M_i} \delta(d_k) \leq 1 \ \forall M_i \qquad \textit{logical constraints} \tag{10}$$

$$V^* > 0 \qquad \textit{gains from exchange} \tag{11}$$

Thus, if $d_i = 0$ then order i is not filled, while if $d_i \neq 0$ then order i is filled up to the scaled amount $F_i \leq d_i \leq 1$. Due to the discrete nature of the packaged bids, there can be cases in which there are gains from exchange, but demand is less than supply ($\Sigma_i \, d_i \bullet q_{ij} < 0$ for some j). In this case, the exchange is made and the untransacted RTCs are retired unused. The set M_i is the set of orders that are "ORed" with order i.

B. The Pricing Rule

Consider the orders charted in Fig. 5. Buyer (B1) has an inflexible order and is willing to pay up to $9 for 3 units and no fewer. Seller (S1) has an inflexible order and is willing to sell 2 units for at least $4. Lastly, seller (S2) is willing to sell 1 unit for at least $4. Surplus is maximized, given the flexibility constraints, if all orders are filled. Specifically, there is no other feasible allocation. However, a per unit competitive price does not exist for this allocation. To see this, observe that at any price above $3 per unit B1 is unwilling to buy units, and at any price below $4 per unit S2 is not be willing to sell any units.

The principles we used in designing the pricing rule were:

(1) revenues sum to zero;
(2) No one pays more (receives less) than they bid (offer).

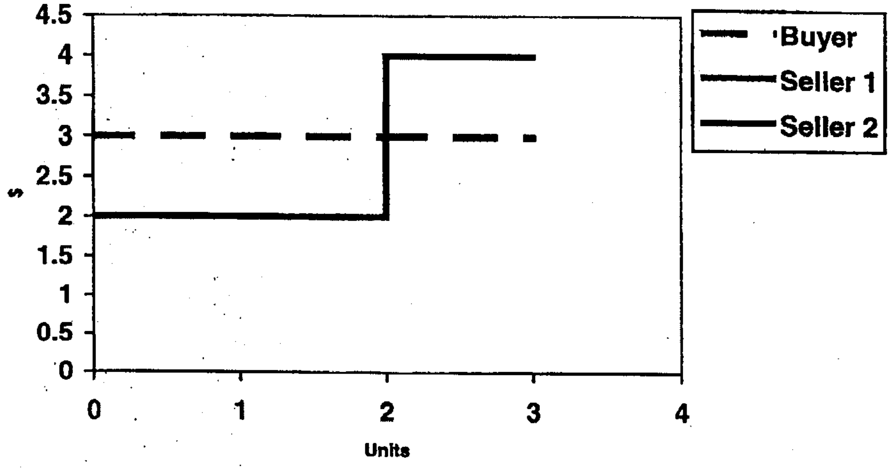


Fig. 5. Non-Existence of Equilibrium for All or Nothing Trades

(3) There are incentives to reveal.

Using these principles, our pricing rule is constructed as follows. After the allocation problem (5)–(9) is solved, there will be three categories of orders:

(i) those orders that were accepted by the allocation;
(ii) those orders that were rejected by the allocation and were *not* part of a logical OR set with an accepted order; and
(iii) those orders that were rejected and were part of a logical OR set with an accepted bid.

If a competitive equilibrium price vector p exists it would satisfy:

$b_i - p\bullet q_i \geq 0$ *for all accepted orders*

$b_i - p\bullet q_i \leq 0$ *for all rejected bids*

$b_{ik} - p\bullet q_{ik} \geq b_{ik*} - p\bullet q_{ik*}$ *if ik is accepted and ik* is ORed with ik*

$p\bullet\Sigma_{i\in A}\, q_i = 0$ *Walras Law*

Unfortunately, the existence and uniqueness of p is not guaranteed. To handle the issue of non-uniqueness we use a "split-the-difference" rule. Specifically, we

(i) Find the competitive equilibrium price that maximizes the net surplus to the buyers (a vector of "low" prices);
(ii) Find the competitive equilibrium price that maximizes the net surplus to the sellers (a vector "high" prices);

(iii) For each commodity, we take the midpoint of (i) and (ii).

In terms of non-existence, we use a "pseudo-competitive equilibrium price". First we ignore rejected and marginal orders.[12] Marginal traders pay/receive what they bid or offer. For the inframarginal traders we check to see if a competitive equilibrium price vector exists for these orders. If it does, that price is used to clear those trades. If such a price does not exist, then the following prices are calculated.[13]

First we take only the accepted orders, i.e. order *i* such that $d^*_i \neq 0$. We then calculate the surplus with the accepted orders where now $d^*_i \in [0,1]$, i.e. the fully flexible allocation. Let V^f denote the surplus from the fully flexible allocation and set $dV = V^f - V^*$, the added surplus from flexibility. Next, we find the vector of prices, one for each order $p^b_i = (p^b_{i1}, \ldots, p^b_{in})$, such that buyer surplus is maximized prices using the accepted orders d^{*i} and such that the budget is balanced including the additional surplus dV and the each buy order pays an amount less than or equal to their bid and each seller receives an amount greater than or equal to their offer. We then to the same process to find the vector of prices $p^s_i = (p_{i1}, \ldots, p^s_{in})$, such that seller surplus is maximized. Next we split the surplus among buyers and sellers by setting $p_{ij} = (p^s_{ij} + p^b_{ij})/2$.

The figure below shows how this price is calculated for our example.

Returning to the example charted in Fig. 5 (reproduced as Fig. 6 above), if traders had been perfectly flexible then the allocation would have B1 buying 2 units from S1 at a competitive price of $2.50 (splitting the surplus) and a total surplus of $2. But since B1 is inflexible, S2 sells a unit at a per unit price above that asked by B1. This "extra marginal" unit is priced so that no surplus is

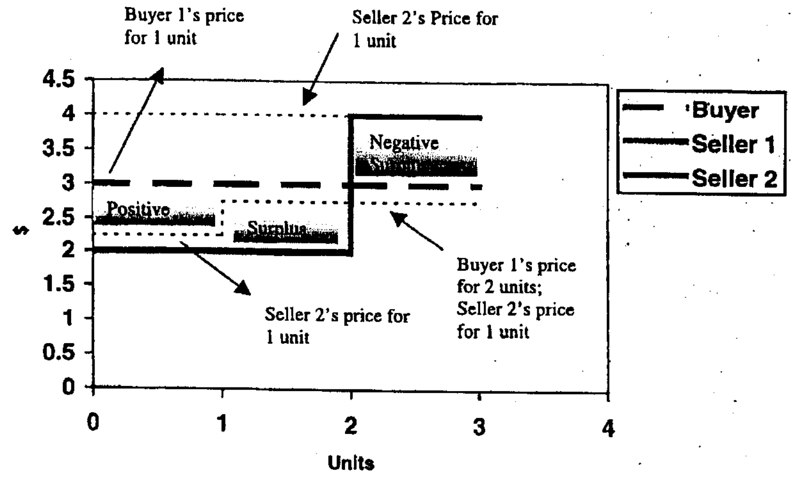


Fig. 6. Price Calculation with All or Nothing Orders.

gained from it. All surplus is assigned to those units that would have been allocated if the traders had been fully flexible.

The pricing mechanism creates a "pseudo-competitive" price pair. In this example, S2 receives \$4 for her one unit, B1 buys one unit at \$3 and 2 units at \$2.75 a unit, and S1 receives \$2.25 a unit. S1 receives \$4.50 for a 50-cent surplus, B1 pays \$8.50 (= \$3 + \$5.50) for a 50-cent surplus, and S2 receives \$4 for a unit for \$0 surplus.

A more complicated example is given in Fig. 7. In this figure, B1 is willing to pay \$9 for 1 unit and B2 is willing to pay \$9 for 3 units. S1 is willing to sell 2 units for \$4 and S2 is willing to sell 2 units for \$8.20. Suppose all of these orders are inflexible. In this case the full allocation gives a surplus of \$5.80.

Since B2 and S2 are marginal S2 receives his ask price for his 2 units and B2 receives his bid for his 2 marginal units. This leaves a deficit of \$2.20 that must be made up by the difference between the price B1 pays for his unit and the price S1 receives for his units. If bids were completely flexible the competitive price would be in the interval between \$2 and \$3, but even if the buyer's and seller's price are at these upper and lower bounds, it would only allow a deficit of \$2 to be covered. Thus, competitive prices cannot be constructed if all bids were flexible. With the constraint that net revenue is zero, buyer and seller prices must be set between \$3.20 and \$9.00, and they only apply to B1's and S1's first unit. The only prices that allow an equal split in surplus between S1 and B1 are \$3.90 for S1 and \$6.10 for B1. In this case B1

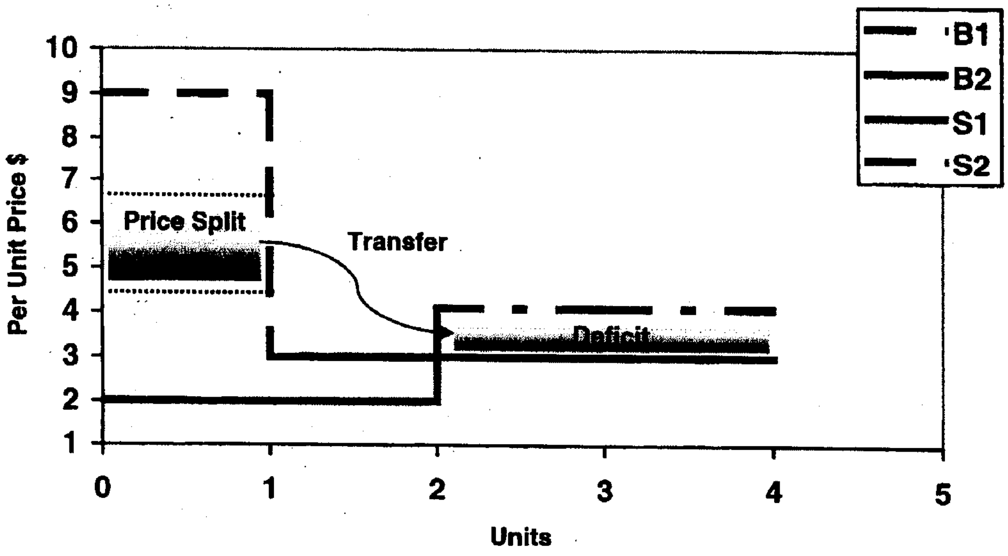


Fig. 7. Price Calculation.

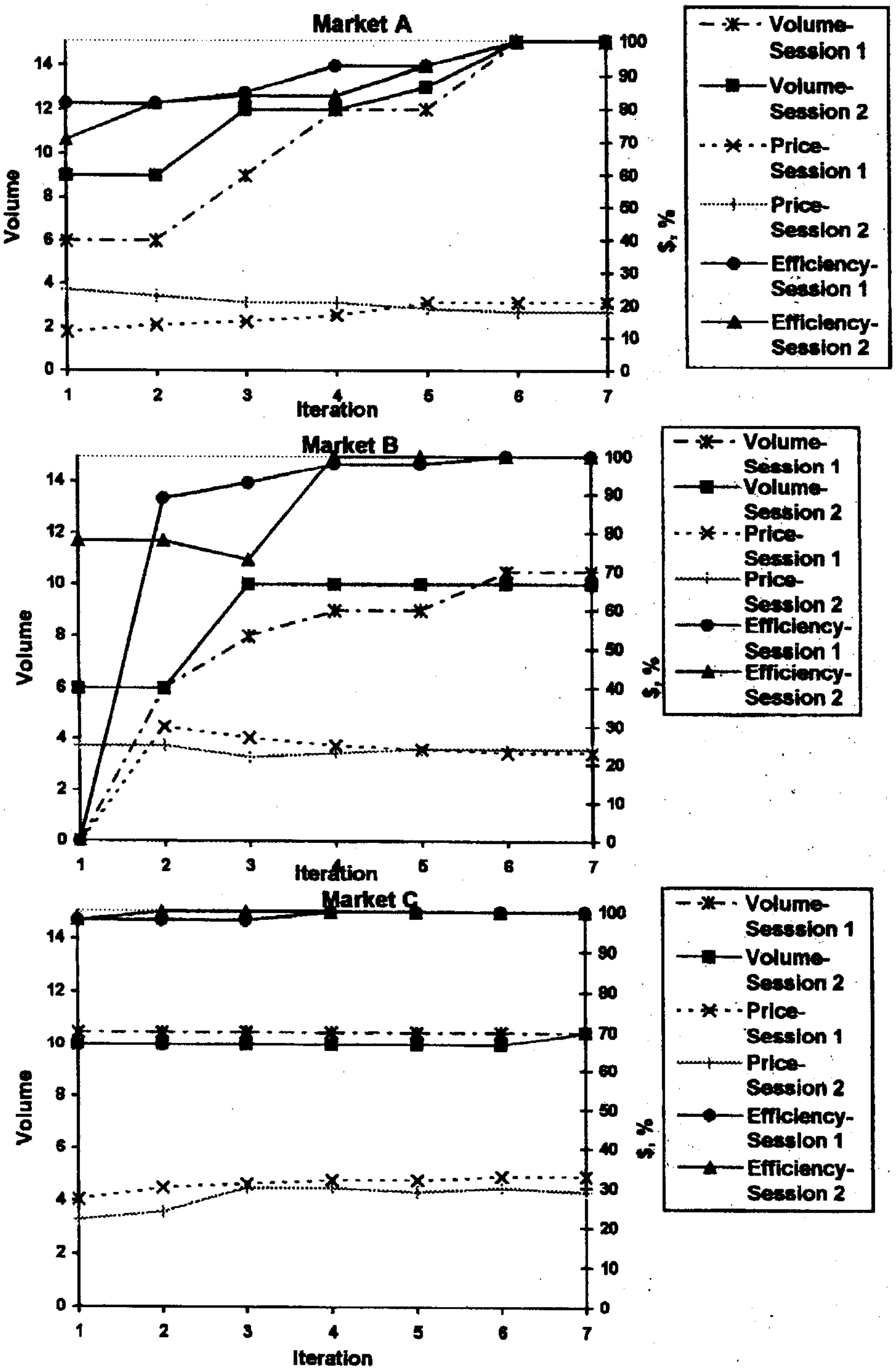


Fig. 8. Superadditive results using TSC.

receives a (\$9 − \$6.10 = \$2.90) surplus and S1 receives a (\$5 + \$2 − \$4 = \$3) surplus. B2 and S2 receive a surplus of \$0.

V. EXPERIMENTAL RESULTS

In this section we describe the results of experiments with this new market design – the Two-Sided Combinatorial auction (TSC). As we did with the UPDA experiments, subjects were recruited from the undergraduate student

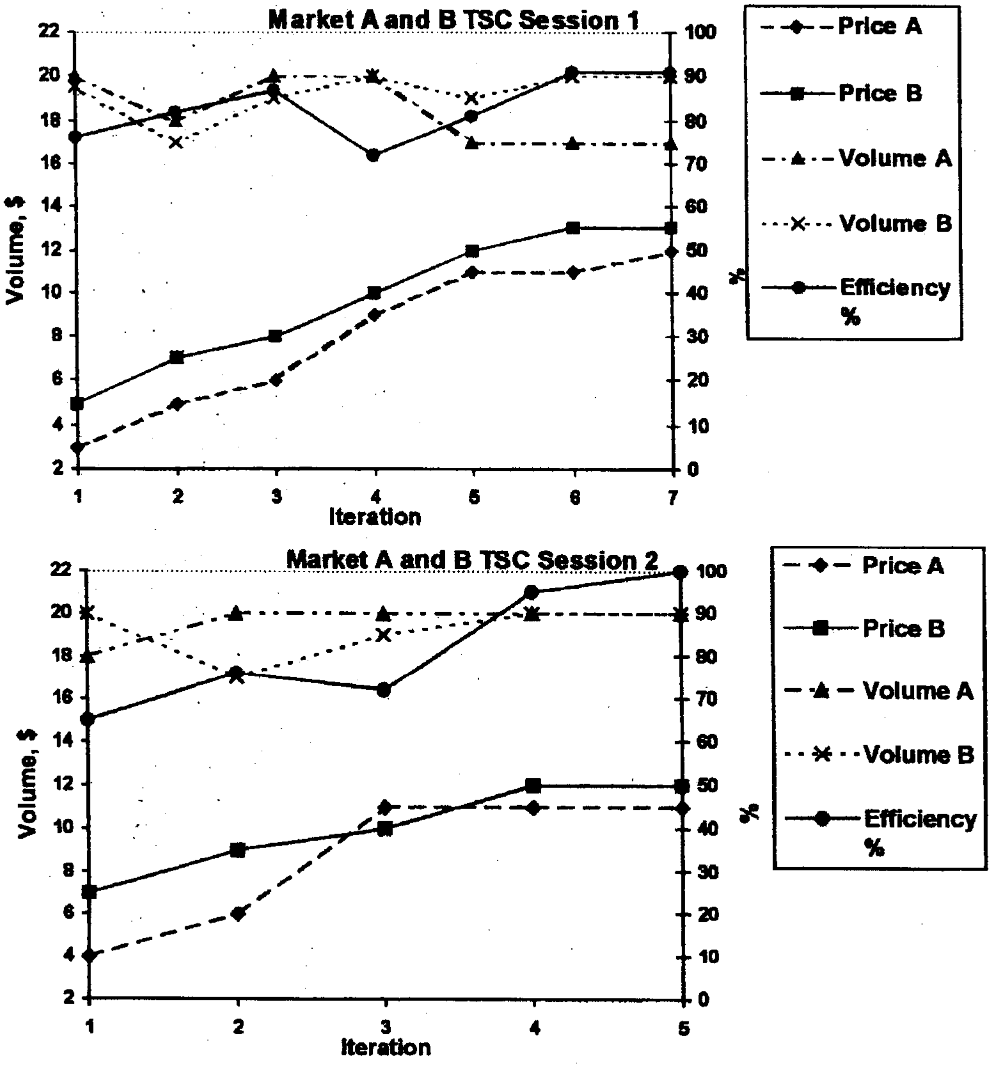


Fig. 9. Strong complement results using TSC.

population at the California Institute of Technology. All the subjects had experience in other market experiments and participated in a 90-minute train session the day prior to the experiments to become familiar with the auction rules and software (they were paid a flat $20 fee for the training session). Each experimental session consisted of a single market instance. Each session was run on a computer network and lasted approximately 1.5 hours. Experimental sessions consisted of 3 commodities that were traded simultaneously. Subjects were assigned randomly to a set of redemption values, costs and endowments. All subjects were paid in cash at the end of the experimental sessions.

The results for the *superadditive* markets are described in Fig. 8. The superadditive values are the same as those found in Fig. 2 that were used with UPDA. The Figure shows the prices, volumes and efficiency for each market. Coincidentally, the auctions both lasted 7 iterations the same as with UPDA. However, unlike UPDA, TSC produced 100% allocations in all sessions and markets. TSC starts off at relatively high efficiencies and reaches over 90% efficiency by iteration 4. Thus, we find:

> *TSC outperforms UPDA in the Superadditive environments. In fact it produces 100% efficient allocations.*

Recall that we conducted experiments with UPDA when the goods were strong complements. Two sessions with the strong complements environment using

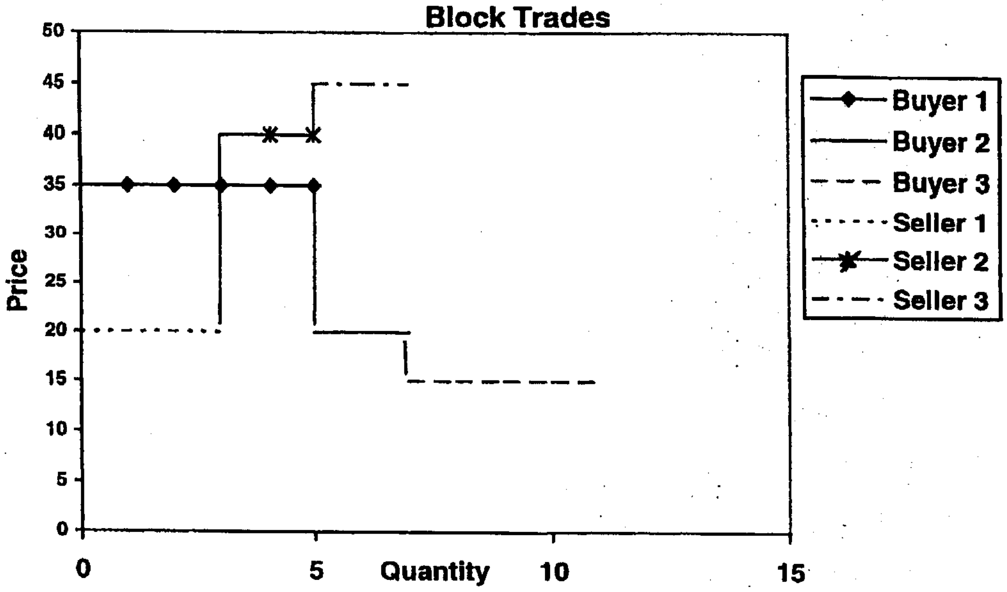


Fig. 10. Block Trade Environment.

TSC were conducted . The results from these session can be found in Fig. 9. Session 1 lasted 7 iterations before the auction closed and session 2 lasted 5 iterations. The results were very similar with outcomes of 90% and 100% efficiencies. Again we find:

> *TSC outperforms UPDA in the Superadditive environments. In fact it produces near 100% efficient allocations.*

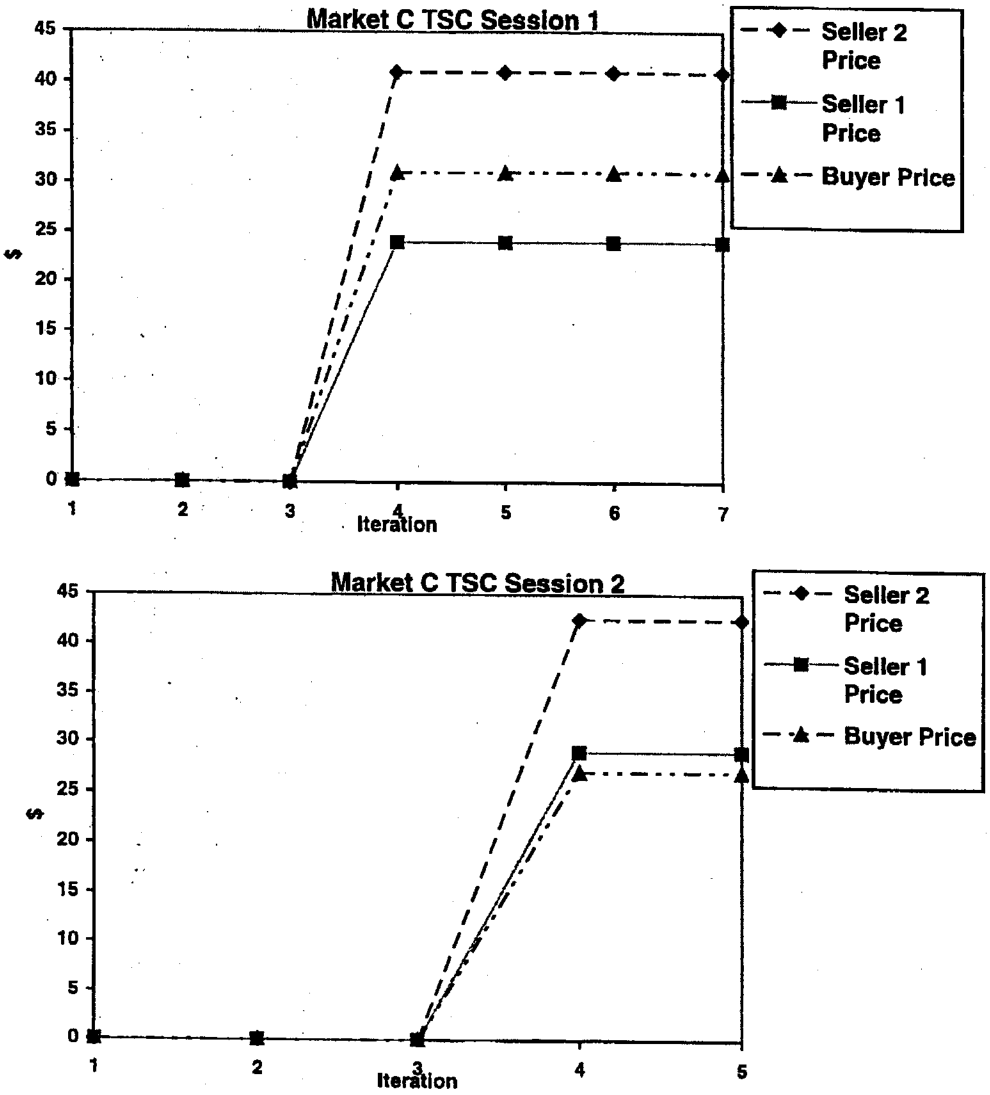


Fig. 11. Block Trade Results using TSC.

In Fig. 1 we provided a case in which all or nothing orders produced a case that is infeasible for UPDA. To see how this type of market is handled by TSC we conducted two sessions with the parameters defined in Fig. 10.The main difference between this environment and the one charted in Fig. 1 is that there is more surplus in the market and cost and values are scaled down. The results of these experiments can be found in Fig. 11. If any trade takes place it must be at 100% efficiency. Thus, in the first few iterations no trade occurs. The prices for the buyer and sellers are charted in the figure for both sessions. The mechanism found a set of prices that split the surplus between seller 1 and the buyer with seller 2 receiving his offer price.

VI. CONCLUSIONS

The design of the emissions trading market for RECLAIM was created to solve an allocation problem when:

(1) The market is thin with a small number of participants;
(2) There is no history of market trades;
(3) Buyers and sellers would like to transact by selling whole portfolios of credits (all or nothing trading, economies of scale, complementary pollution emissions requirements, etc.)

The market that was designed to allow buyers and sellers to tailor their orders so that entire portfolios of credits can be traded. The main incentive feature of the market is the pricing rule that calculates prices from marginal traders and thus inframarginal traders have an incentive to reveal values and costs. The combinatorial nature of the allocation process allows the inframarginal portfolio traders to reveal without the danger of financial exposure.

The experiments conducted in this paper show that this market design allocates units at near 100% efficiency, which a Uniform Price Auction cannot come close to producing.

NOTES

1. The program also has two compliance cycles, January through December and July through June. These overlapping cycles were created to increase liquidity and decrease price volatility at year-end (see Carlson et al. (1993)).

2. Uncertainty about future regulatory decisions that will impact the value of the emission credits is one of the major reasons why there have been such a small number of trades in the EPA's SO_2 program and California's ERC program. Burtaw (1996) points out that in the SO_2 market, state utility regulatory commissions have not provided any rules for treating the cost recovery of SO_2 trades and are basically considered

current expenses with all cost savings passed through to ratepayers. Thus, there is little incentive for the regulated utility to reduce costs through sales and purchases of credits. In addition, state legislatures have prohibitions on trades that might undermine local economic activity. Hahn and Hester (1989) note that firms face considerable uncertainty in how regulators will determine their future emission reductions. Thus, a buyer will not know whether the seller will have his credits recognized by the regulators as a real emissions reduction. Thus, there is no guarantee that credits will be delivered from a trade.

3. The allocations were partially based on past emission levels and "political considerations". In addition, the EPA retained a small fraction of allowances that it sold in special auctions and direct sales.

4. To find out more about the actual market, visit the website *http://www.ace-mkt.com*

5. For example, a firm's production plans might require it to emit 1 million pounds of NOx per year for 5 years using current technology. If the firm has an 800,000-pound cap on NOx emissions, it will not be able to meet its production goals and stay within its environmental constraints. To meet these goals the firm may have a number of options:

- Purchase pollution abatement equipment that will reduce 200,000 pounds of NOx per year for $1million.
- Purchase 200,000 RTCs for each of the 5 years.
- Purchase 100,000 RTCs for each of the 5 years and purchase pollution abatement equipment that will reduce 100,000 pounds of NOx per year for $600,000.
- Trade current year RTCs for future RTCs (switching production).

The option that is most cost effective for the firm will depend on the cost of RTCs. The firm would like to be able to purchase 5 years' of RTCs as one package. If the firm were to purchase the first year credits and then find that the prices of the last year credits exceed the cost of the investment in equipment, the firm might find that it has not acted in the most cost-effective way.

6. The need for markets to clear simultaneously has been shown to be an important feature in other auctions. For example, in the FCC spectrum auctions, the major design feature utilized is that all the licenses within a band had to be auctioned-off simultaneously so that bidders could know their substitution possibilities (see Milgrom (2000)).

7. If person i only submits $u < m$ bids, then P^z_{ij}, $Q^z_{ij} = 0$ for $z > u$.

8. In Market C the only seller was the experimenter, who offered all 10 units at a zero price.

9. Recall that a main difference in Market C relative to Markets A and B was that there were no sellers in Market C. The Experimenter offered all ten units at a zero price.

10. This does not mean that the mechanism does not have transaction fees that are remuneration to the market designers and operators. This market imposes a 3% transaction free for each trade based on the value of the trade. This transaction fee is automatically taken into account in the bids and offers submitted in the market. We needed to imbed the transaction fee into the market due to the requirement that individuals could not sell short or trade on margin. In order to guarantee RTC delivery and funds transfers from trades, participants are required to escrow their credits and

funds prior to the market opening so that trades are ensured. Since transaction fees are part of the transfers that had to be incorporated into the bids no deficits are possible. This is one of many practical issues we needed to address in the market design process. For more details on the market design specifications see Ishikida et al. (1999).

11. In the actual market, there is a maximum of 5 iterations and a minimum of 2 iterations per auction.

12. This allows for the possibility that $b_i - p \cdot q_i > 0$ for unaccepted orders.

13. For a more detailed and formal representation of this pricing rule see Ishikida et al. (1999).

REFERENCES

Atkinson, & Tietenberg (1991). Market Failure in Incentive-Based Regulation: the Case of Emissions Trading, *Journal of Environmental Economics and Management, 21*, 17–31.

Banks, J. S., Ledyard, J. O., & Porter, D. P. (1989). Allocating uncertain and unresponsive resources: an experimental approach, *RAND Journal of Economics*, *20*(1), (Spring).

Burtaw, D. (1994). The SO_2 emissions trading program; cost savings without allowance trades, *Contemporary Economic Policy, 14*, 79–94.

Campbell, J., LaMaster, S., Smith, V., & Van Boening, M. (1991). Off-floor Trading, Disintegration and the Bid-Ask Spread in Experimental Markets. *The Journal of Business*, *64*, 495–522.

Carlson, D., Forman, C., Ledyard, J., Olmstead, N., Plott, C., Porter, D., & Sholtz, A. (1993). *An Analysis and Recommendation for the Terms of the RECLAIM Trading Credit*, Pacific Stock Exchange Technical Report, April 27, 1993.

Cason, T. (1995). An experimental investigation of the seller's incentives in EPA's emission trading auction, *American Economic Review, 85*, 905–922.

Cason T., & Plott, C. (1996). EPA's new emissions trading mechanism: a laboratory evaluation, *Journal of Environmental Economics and Management, 30*, 133–160.

Davis, D. D., & Holt, C. A. (1993). *Experimental Economics*, Princeton University Press, Princeton, New Jersey.

Friedman, D. (1993). How trading institutions affect financial market performance: some laboratory evidence, *Economic Inquiry, 31*(3), 410–435.

Hahn, R., & Hester, G. (1989). Where did all the markets go? *Yale Journal on Regulation, 6*, 109–153.

Ishikida, T., Ledyard, J., Olson, M., & Porter, D. (1999). A Two-Sided Packaging Mechanism for Allocating Pollution Emission Credits: Results from Southern California's RECLAIM Market, mimeo, California Institute of Technology.

Joskow, P., Schmalensee, R., & Bailey, E. (1998). The Market for Sulfur Dioxide Emissions, *American Economic Review, 88*(4), 669–685.

Ledyard, J. (1993). The design of coordination mechanisms and organizational computing, *Journal of Organizational Computing, 3*(1), 121–134.

Liu, Y-J. (1992). Auction mechanisms and information structure: an experimental study of information aggregation in security markets, Mimeo. National Chung Cheng University, Taiwan.

McCabe, K. A., Rassenti, S. J., & Smith, V. L. (1993). Designing a uniform-price double-auction: an experimental evaluation. In: D. Friedman & J. Rust (Eds.), *The Double Auction Market*, *SFI Studies in the Sciences of Complexity, Proc.* Vol. XIV (pp. 307–332). Addison-Wesley.

Milgrom, P. (2000). Putting auction theory to work: the simultaneous ascending auction, *Journal of Political Economy, 1081*(2), 245–272.

Montgomery, D. (1972). Markets in licenses and efficient pollution control programs, *Journal of Economic Theory,* 395–418.

Plott, C. (1994). Market architectures, institutional landscapes, and testbed experiments, *Economic Theory, 4,* 3–10.

Skiena, S. (1997). *The Algorithm Design Manual,* Springer-Verlag, New York.

Smith, V. L., Williams, A. W., Bratton, W. K., & Vannoni, M. G. (1982). Competitive market institutions: double auctions versus sealed bid – offer auctions, *American Economic Review, 72,* 58–77.

Van Boening, M. (1991). *Call Versus Continuous Auctions: An Experimental Study of Market Organization.* Dissertation, University of Arizona.

Van Boening, M., & Wilcox, M. (1996). Avoidable cost: ride a double auction roller coaster, *American Economic Review, 86,* 461–477.

APPENDIX A

Environment Values

Superadditive Values*

Subject	1	2	3	4	5	6	7	8
Endowment Market A	0	0	0	0	0	5	5	5
Value Unit1	20	20	20	20	20	15	15	15
Unit 2	35	30	30	25	20	12	12	12
Unit 3	45	40	35	30	25	11	11	11
Unit 4	0	0	0	0	0	0	0	0
Endowment Market B	0	0	0	0	3	3	2	2
Value Unit 1	27	13	7	5	25	25	25	27
Unit 2	28	15	10	9	21	19	13	15
Unit 3	29	17	13	13	18	17	10	10
Unit 4	30	19	16	17	7	7	7	7
Unit 5	31	21	19	21	5	5	5	5
Unit 6	32	23	21	25	0	0	0	0
Unit 7	33	25	24	29	0	0	0	0
Unit 8	34	27	27	33	0	0	0	0
Unit 9	35	29	30	37	0	0	0	0
Unit 10	36	31	33	40	0	0	0	0
Endowment Market C	0	0	0	0	0	0	0	0
Value Unit 1	50	50	50	30	30	35	35	35
Unit 2	58	58	58	55	55	60	60	60
Unit 3	60	60	60	60	60	65	65	65

* Values are in experimental currency – francs; 1 franc = $.50

Strong Complements Values*

Bidder ID	Amount of A	Amount of B	Value	Bidder ID	Amount of A	Amount of B	Value
1	4	3	100	2	3	6	125
1	7	3	175	2	3	10	150
1	12	3	250	2	3	14	175
1	4	9	150	2	9	6	175
1	7	9	225	2	9	10	190
1	12	9	325	2	9	14	200
1	4	13	175	2	15	6	200
1	7	13	250	2	15	10	225
1	12	13	335	2	15	14	250
3	3	2	75	4	6	8	100
3	3	4	100	4	6	10	150
3	3	9	125	4	6	12	200
3	5	2	100	4	8	8	150
3	5	4	200	4	8	10	200
3	5	9	225	4	8	12	275
3	12	2	175	4	12	8	175
3	12	4	250	4	12	10	250
3	12	9	275	4	12	12	300
5	6	7	175	6	7	7	75
5	6	10	225	6	7	9	150
5	6	13	250	6	7	11	175
5	9	7	225	6	9	7	125
5	9	10	275	6	9	9	175
5	9	13	300	6	9	11	200
5	12	7	250	6	11	7	150
5	12	10	300	6	11	9	200
5	12	13	325	6	11	11	225

* Values are in experimental currency – francs; 1 franc = $.10

APPENDIX B

Instructions for TSC

You are about to participate in an economic experiment in which you will earn money based on the decisions you make. All earnings you make in the experiment are yours to keep. Here are some important features of our experiment:

- All accounting (values, prices and costs) will be stated in francs. Each franc you earn can be converted into U.S. currency at the rate specified on your **Opportunities Sheet** (see below).
- We will be conducting a Market in which you can buy and/or sell the items found on your Opportunities Sheet.
- The Market will consist of **Rounds** in which you will make decisions. *You only earn francs when the Market closes, not for each Round of the Market.*
- When the Market closes you will fill out your **Accounting Sheet** and a monitor will verify your earnings and accounting.

The Opportunities Sheet

Your Opportunities Sheet is private information; you should not reveal it to anyone. This sheet lists items (A, B, C) that you can buy or sell. Below you will find a sample sheet for participant 10 who has a franc conversion rate of ___ francs to a dollar. In addition, this participant must pay ___ francs to use the sheet. This participant has an inventory of ___ units of item A, ___ units of B, and ___ units of C. Each participant is free to sell up to, but not beyond, the number of units they have in their inventory. Under each item there is a value listed in francs for the number of units; you finally hold when the market closes. The last column in the table lists the additional value of getting the particular package combination of the items. For example, if the participant were to get 1 unit of B she would receive ___ francs; if she were to get two units of B she would get an additional ___ francs or a total of ___ francs. If the participant obtained ___ units of A and ___ units of C she would get ___ francs in value.

Earnings for a period are equal to the sum of the values obtained plus revenue from units sold in the market minus the cost of the units bought in the market. For example, if at the end of the period the participant sold 3 units of C at ___ francs each; and bought 2 units of B at a price of ___ her earnings for the period would be:

OPPORTUNITIES SHEET

Period _____

Participant _____ Conversion Rate _____ francs = _____ Dollars

Cost __________

A Market		B Market		C Market		Package
Units	**Value**	**Units**	**Value**	**Units**	**Value**	**Value**

INITIAL INVENTORY

A = ______________ **B =** ______________ **C =** ______________

Accounting Sheet

When the market closes you must fill out an accounting statement for the period. Attached to these instructions you will find an accounting sheet. Just fill in the lines of the form and make the proper additions and subtractions.

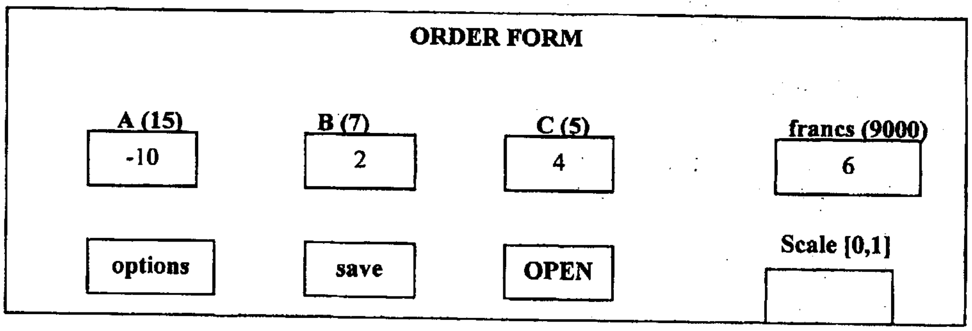
How the Market Trading Systems Works

Submitting an Order

The market is divided into rounds in which you can submit orders to the market. An **order** is a listing of the items A, B and C along with a franc amount. A minus entry means that you want to **sell** the specified number of units and a positive entry means you want to **buy** the specified number of units. In the franc position a negative amount (let us call it your **sell offer**) implies you want to receive the specified amount or more; a positive sign (let us call this your **buy offer**) implies you are willing to pay that amount or less. Below you will find an example. There are four upper boxes in the **ORDER FORM** that lists each of the items A, B and C along with a box for francs. Next to each item in parentheses, is the amount of units you are allowed to sell in the market and the amount of francs you have in your credit line. In the example **order form** below, the order is to sell 10 units of A and buy 2 units of B and 4 units of C. The entire package is offered with a buy offer of 6 francs or less.

There are several features of the above order that are important:

- The order is for a package of items. It is an *all or nothing* order in that the entire amounts –10A, 4B and 2C must be filled if the order is to be a valid transaction.
- The franc amount is a total package price offered, not a per unit price for a specific item.
- A buy offer is an *upper bound* on what you are willing to pay. A sell offer is a *lower bound* on the amount of francs you are willing to receive for the order.



ORDER FORM

A (15)	B (7)	C (5)	francs (9000)
-10	2	4	6

Tailoring Your Order

Your order can be customized in three ways that may assist you in obtaining a desired transaction.

Scaleable Orders. Recall that the order placed in the above example was all or nothing. You can request that the order be less restrictive. You do this by submitting a scale number between 0 and 1 that you would be willing to scale your order to get it accepted. The best way to explain this feature is by an example. In the order form below a scale of 0.5 is submitted with the order. This means that you want to sell at least 5 units of A and buy at least 1 unit of B and 2 units of C for a package price of 3 francs. That is, all units are scaled by one-half. The scale also means that you are willing to scale up your order from this minimum in all dimensions up to the full order.

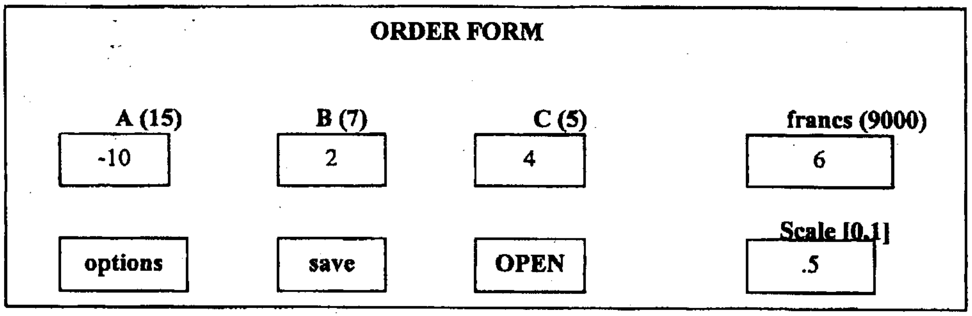
Contingent Orders. If you submit more than one order you can tie your orders together. When two orders are tied together they become linked so that if one of them is accepted the other *will not* be accepted. In the example below, the bidder has two orders (Ids 4 and 5) that he has already constructed and wants his current order of –10A, 2B, 4C at a buy offer of 6 to be contingent with orders 4 and 5. He does this by tying them together with the same **Con Id.** This means that at most one of the orders with the same Con Id will be filled.

Open Orders. Unless you specify otherwise, the orders you submit in a round will be closed, i.e. no other participant can view what you send to the market during the round. If you would like others to see what you send to the market before the round finishes you can send the order to the **open book**. Orders in the open book can be viewed by all participants and will be part of the orders submitted to the market for the round.

Order Restrictions

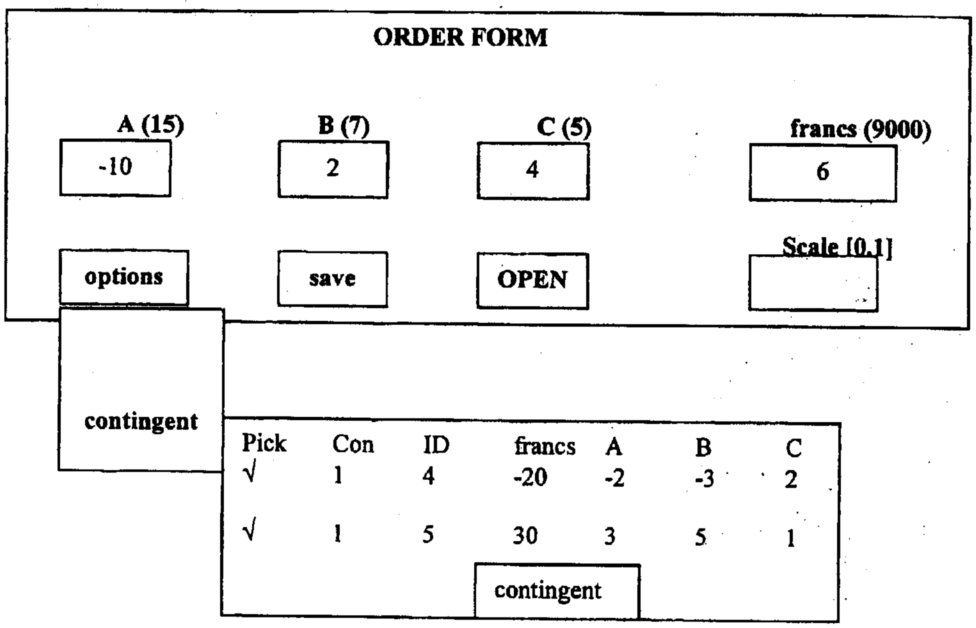
The following three constraints will be placed on your orders:

- Your order can be submitted to the market if it does not violate your credit line and you try to sell more units than you have in your account. The



ORDER FORM

A (15): -10
B (7): 2
C (5): 4
francs (9000): 6
Scale [0,1]: .5



ORDER FORM

A (15)	**B (7)**	**C (5)**	**francs (9000)**
-10	2	4	6

Scale [0,1]

Pick	Con	ID	francs	A	B	C
√	1	4	-20	-2	-3	2
√	1	5	30	3	5	1

constraint on your orders is cumulative. That is, each time an order is submitted your account is reduced by the amounts in that order. Sells count against the item account and buy offers are subtracted from your franc account. For contingent orders, we will use the maximum amount of each sell item and buy offer among the tied orders to debt your account. However, sell orders do not increase your franc account and buy orders do not increase your unit accounts.

- One other important restriction on the accepted orders in a round is that all transactions will be in whole numbers, i.e. units are transacted in increments of size 1. Thus, all bid amounts must be in integer amounts and all transactions will be made in integer amounts.
- Only previous round "accepted" orders and open book orders can be changed (they must have a higher buy price or lower sell price). This will be discussed in more detail later.

ROUND Results

Orders submitted in a round will be placed into a program that finds which orders are standing for the round using the following rules:

(1) The accepted orders do not violate the order parameters.

(2) The total amount sold is greater than or equal to the total amount purchased.

(3) The difference between the buy offers and the sell offers from the accepted orders that obey 1 and 2 is as large, net of transaction fees, as we can find. We will call this amount the market **surplus**.

The set of accepted orders from the above rules become the **Standing Orders** for the market. Standing orders in a round are automatically submitted to the next round. Participants holding standing orders or open book orders may increase the offer price on the order, and may include the order in a contingent order. Participants can also submit new orders each round.

Market buy prices and market sell prices will be calculated each period. The market price of an order is then (buy prices * amounts bought) – (sell prices * amounts sold). Prices will be chosen, if possible, so that

(1) The market buy and market sell prices are the same.
(2) The market price of each accepted (rejected) order, is less than (greater than) or equal to its offer amount.

The amount you will pay (receive if the market price is negative) for an accepted order if the market were to close in that round is:

Your Payment = (market buy prices*amounts you buy)–(market sell prices*amounts you sell)

Round Result Information

When the round ends, we will display aggregate results of the round. Specifically, we will provide participants with the following informational display

Round	Current Surplus	Change in Surplus	Total Volume	Change in Total Volume
3	100	20	10	5

	A	B	C
market buy price	2.0	1.0	2.0
market sell price	1.5	0.5	1.75
units traded	3.0	3.0	4.0
units available	10.0	10.0	10.0
sell units offered	5.0	5.0	10.0
buy units offered	10.0	10.0	10.0

You will also see the orders that you had accepted along with your prices. After the round starts you will be able to see the open book orders.

STOPPING the Period

The auction period will end after round 2 if surplus or volume does not increase by 5% over previous round surplus or volume.

Accounting Sheet Period _____

Participant _____ SSN ____________ E-mail ____________________ Phone _____________

| Final Accepted Orders |

Round/ Order No.	A	B	C	Value	A*your price	B*your price	C*your price	Cost	Discount	Earnings
Initial Holdings										
Final Holdings					-	-	-	-	+	=

Discount Accounting

Order Amount + + = x =

Period	Round/ Order No.	A	B	C	Your Price*A	Your Price*B	Your Price*C	Total	Rate %	Fee	Discount